Dynamics of competition between collectivity and noise in the stock market

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Abstract

Detailed study of the financial empirical correlation matrix of the 30 companies comprised by DAX within the period of the last 11 years, using the time-window of 30 trading days, is presented. This allows to clearly identify a nontrivial time-dependence of the resulting correlations. In addition, as a rule, the draw downs are always accompanied by a sizable separation of one strong collective eigenstate of the correlation matrix which, at the same time, reduces the variance of the noise states. The opposite applies to draw ups. In this case the dynamics spreads more uniformly over the eigenstates which results in an increase of the total information entropy.

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Studying correlations among various financial assets is of great interest both for practical as well as for fundamental reasons. Practical aspects relate for instance to the theory of optimal portfolios and risk management [1]. The fundamental interest, on the other hand, results from the fact that such study may shed more light on the universal aspects of evolution of complex systems. Recent study [2, 3] of the related problems in the context of the stock market show that majority of eigenvalues in the spectrum of the correlation matrix agree very well with the universal predictions of random matrix theory. Locations of some of the eigenvalues differ however from these predictions and thus identify certain system-specific, non-random properties of the system. The corresponding eigenvalues thus carry most of the information about the system. The above studies have however a global in time character and do not account for a possible change of correlations on shorter time-scales.

For a set of N assets labelled with i and represented by the time-series of price changes $\delta x_i(t)$ of length T one forms a $N \times T$ rectangular matrix $\mathbf{M}$. Then, the correlation matrix is defined as

$$\mathbf{C} = \frac{1}{T}\mathbf{M}\mathbf{M}^{\mathbf{T}}. \tag{1}$$

In order not to artificially reduce the rank of this matrix, T needs to be at least equal to N. This sets the lowest limit on a time-window which can be used to study the time-dependence of correlations. One of the best examples where T can be set relatively low, and thus efficiently allow to get some time-dependent picture of correlations versus the global market index, is provided by DAX (Deutsche Aktienindex). It represents a matured, relatively independent market, whose behavior is well reflected by $N = 30$ companies defining this index. During the period studied here it displays the whole variety of behaviors like stagnancy, booms, including a pattern of self-similar log-periodic structures [7], and crashes.

The present study is based on daily variation of all $N = 30$ assets of the DAX during the years 1988-1999. When calculating the covariance matrix the average value of the price changes is subtracted off and then their values rescaled so that $\sigma^2 = \langle \delta x_i^2 \rangle = 1$. The time-interval T is set to 30 and continuously moved over the whole period.

That the character of correlations may significantly vary in time is indicated in Fig. 1 which displays some typical distributions of matrix elements of $\mathbf{C}$ in three different cases: (i) an average over all time-intervals of length $T = 30$, (ii) for a single $T = 30$ time-interval which ends on November 25, 1997, (iii) for a single $T = 30$ time-interval which ends on April 7, 1998. Clearly, in all the cases the distributions are Gaussian-like but their variance and location is significantly different. In fact, a distribution of this type about prescribes the structure of the corresponding eigenspectrum. The point is that to a first approximation any of such matrices can be represented as

$$\mathbf{C} = \mathbf{G} + \gamma \mathbf{U}, \tag{2}$$

where $\mathbf{G}$ is a Gaussian centered at zero and $\mathbf{U}$ is a matrix whose all entries are equal to unity and $0 \leq \gamma \leq 1$. The rank of matrix $\mathbf{U}$ is one, therefore the second term alone in the above equation generates only one nonzero eigenvalue of magnitude γ. As the expansion coefficients of this particular state are all equal this assigns a maximum of collectivity to such a state. If γ is significantly larger than zero the structure of the matrix $\mathbf{U}$ is dominated by the second term in (2) and an anticipated result is one collective state with large eigenvalue. Since in this case $\mathbf{G}$ can be considered only a 'noise' correction to $\gamma\mathbf{U}$ all the other states are connected with small eigenvalues. The above provides an alternative potential mechanism for emergence of collectivity out of randomness to the one taking place in finite interacting Fermi systems. There, a reduction of dimensionality of a leading component in the Hamiltonian matrix is associated with appearance of more substantial tails in the distribution of large matrix elements as compared to an ensemble of random matrices [7].

A relatively small number of stocks comprised by DAX makes somewhat difficult a full statistical analysis of the 'noise' term. Its properties, analogous to the predictions of the Gaussian orthogonal ensemble (GOE) [8] of random matrices, are however already well established in the recent literature [2, 3]. In the present case instead, one can trace a possible non stationarity in the location of eigenvalues with a comparatively good time-resolution. The corresponding central result of our paper is displayed in Fig. 2. As it is clearly seen, the draw ups and the draw downs of the global index (DAX), respectively, are governed by dynamics of significantly distinct nature. The draw downs are always dominated by one strongly collective eigenstate with large eigenvalue. Such a state thus exhausts a dominant fraction of the total portfolio variance

$$\sigma_P = \sum_{i,j}^{N} p_i C_{ij} p_j, \tag{3}$$

where p_i expresses a relative amount of capital invested in the asset i, and C_{ij} are the entries of the covariance matrix $\mathbf{C}$ [1, 5]. (This becomes obvious by transforming $\mathbf{C}$ to its eigenbasis.) The more dramatic the fall is the more pronounced is this effect. At the same time, by conservation of the trace of $\mathbf{C}$, the remaining eigenvalues (representing some less risky portfolios) are compressed in the region close to zero. In a formal sense, this effect is reminiscent of the slaving principle of synergetics [9]: one state takes the entire collectivity by enslaving all the others.

The opposite applies to draw ups. Their onset is always accompanied by a sizable restructuring of eigenvalues towards a more uniform distribution. The related principal effect is that the largest eigenvalue moves down which is compensated by a simultaneous elevation of lower eigenvalues. At one instant of time (mid 1996), which marks the beginning of the long-term boom, the two largest eigenvalues become almost degenerate. Based on these results a general statement that an increase on the market involves more competition than a decrease seems appropriate since in the former case the total variance is more democratically distributed among eigenstates of the correlation matrix. In other words, the increase on the stock market, at least in the presently studied case, never involves parallel uniform increase of prices of all the participating companies as it happens during decreases.

Such a conclusion is also indicated by the information entropy:

$$I_k = \sum_{l=1}^{30} -(u_{kl})^2 \ln(u_{kl})^2, \tag{4}$$

where u_{kl} (here $l = 1, ..., 30$) are the components of eigenvector k. Its GOE limit [10] is

$$I^{GOE} = \psi(N/2+1) - \psi(3/2) \tag{5}$$

where ψ is the digamma function and N, in the present study, corresponds to the number of stocks. For $N = 30$ we thus have $I_k^{GOE} \approx 2.67$ and for the uniformly distributed components ($u_{kl}^2 = 1/30$) $I_k^{uni} = 3.4$. These two limits are to be related to the information entropies displayed in the lower panel of Fig. 2. As one can see, it happens only during decreases that the information entropy approaches the limit of uniform distribution for the upper, most collective state. Otherwise this state becomes somewhat more localized. The information entropy of the 'noise' states on average about agrees with I^{GOE}, though, a more careful inspection shows systematic and consistent deviations, going in opposite directions during increases and decreases, respectively.

In quantitative terms this effect can easily be deduced by looking at the total information entropy

$$I_{tot} = \sum_{k=1}^{30} I_k \tag{6}$$

shown in Fig. 3. On average it reveals a visible tendency of moving in opposite direction relative to the information entropy I_1 of the most collective state, even though I_1 is included in I_{tot}. This result is very interesting and even intriguing in itself. The market draw ups are accompanied by increase of the total information entropy while draw downs are associated with its decrease. At first glance such a behavior and, especially, the entropy decreases accompanying such turmoils as crashes may look somewhat embarrassing. A possible reason for this effect might be the fact that prices and related quantities reflect only a part of the market world. There is also an environment with which any market constantly interacts and which easily may absorb a corresponding portion of entropy. Actually, the turmoils accompanying crashes are visible more in the market environment than in the genuine market parameters.

Since the structure of the covariance matrix is influenced by measurement noise more for short time-series than for the long ones, a question which needs to be answered is to what extent our conclusions are stable with respect to the length T of the price time-series. Of course, the specific values of the quantities considered do depend on T but the main effect of increasing it is to smear them out in time. The global tendencies, of interest for the present paper, remain however unchanged. An example is shown in Fig. 4 which displays the structure of eigenspectrum of the covariance matrix for several values of T.

In summary, the present study discloses several interesting novel facts about the dynamics of financial evolution. These empirical facts, interpreted in terms of the coexistence of collectivity and noise in correlations among the financial assets, provide arguments for distinct nature of the mechanism governing financial increases and decreases, respectively, even though such correlations on average are largely compatible with the random matrix theory predictions. The structure of eigenspectrum of the correlation matrix and the information entropy arguments point to increases as those phenomena which internally involve more diversity and competition as compared to decreases. It seems likely that such characteristics may apply to the dynamics of evolution of other complex systems as well.

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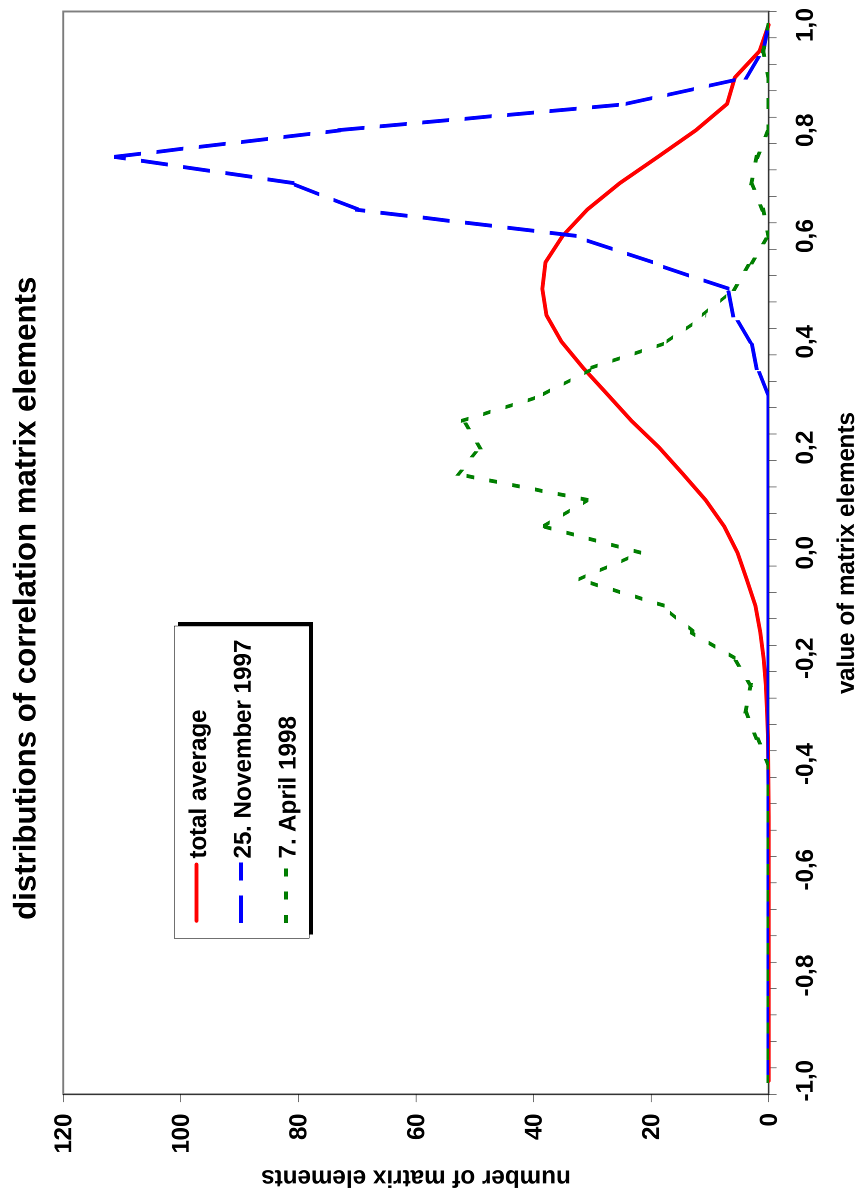

Figure 1: *Distributions of matrix elements of the correlation matrix* $\mathbf{C}$ *calculated from daily price variation of all* $N = 30$ *companies comprised by DAX in three different cases: (i) an average over all time-intervals of length* $T = 30$ *during the period 1988-1999. (ii) for a single* $T = 30$ *time-interval which ends on November 25, 1997, (iii) for a single* $T = 30$ *time-interval which ends on April 7, 1998.*

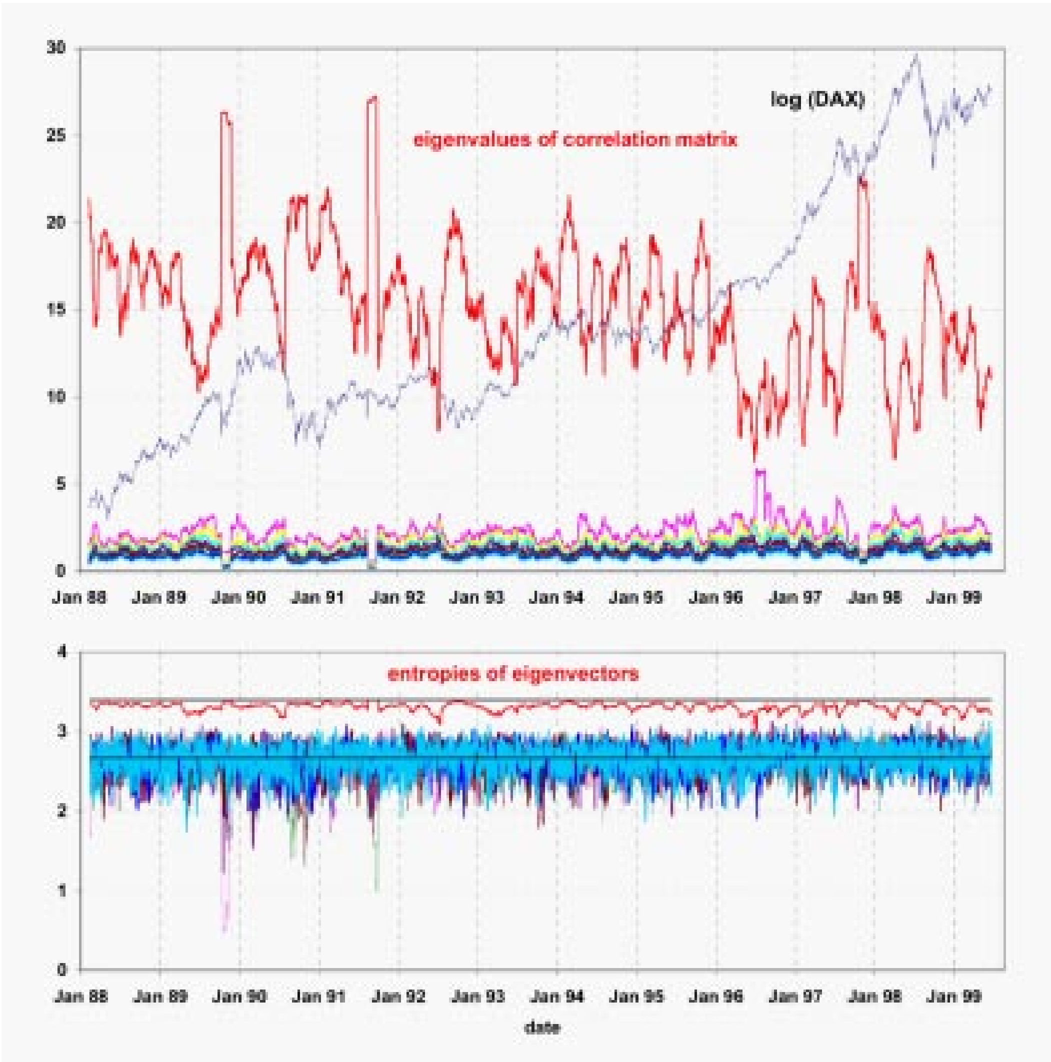

Figure 2: *Time-dependence of the 10 largest eigenvalues (upper panel) and of the related spectrum of information entropies (lower panel) corresponding to the DAX correlation matrix* $\mathbf{C}$ *calculated from the time-series of daily price changes in the interval of* $T = 30$ *past days, during the years 1988-1999. The DAX time-variation (represented by its logarithm) during the same period is displayed in the upper panel. The two solid horizontal lines in the lower panel indicate the two reference limit values,* $I^{GOE} = 2.67$ *and* $I^{uni} = 3.4$*, of the information entropy.*

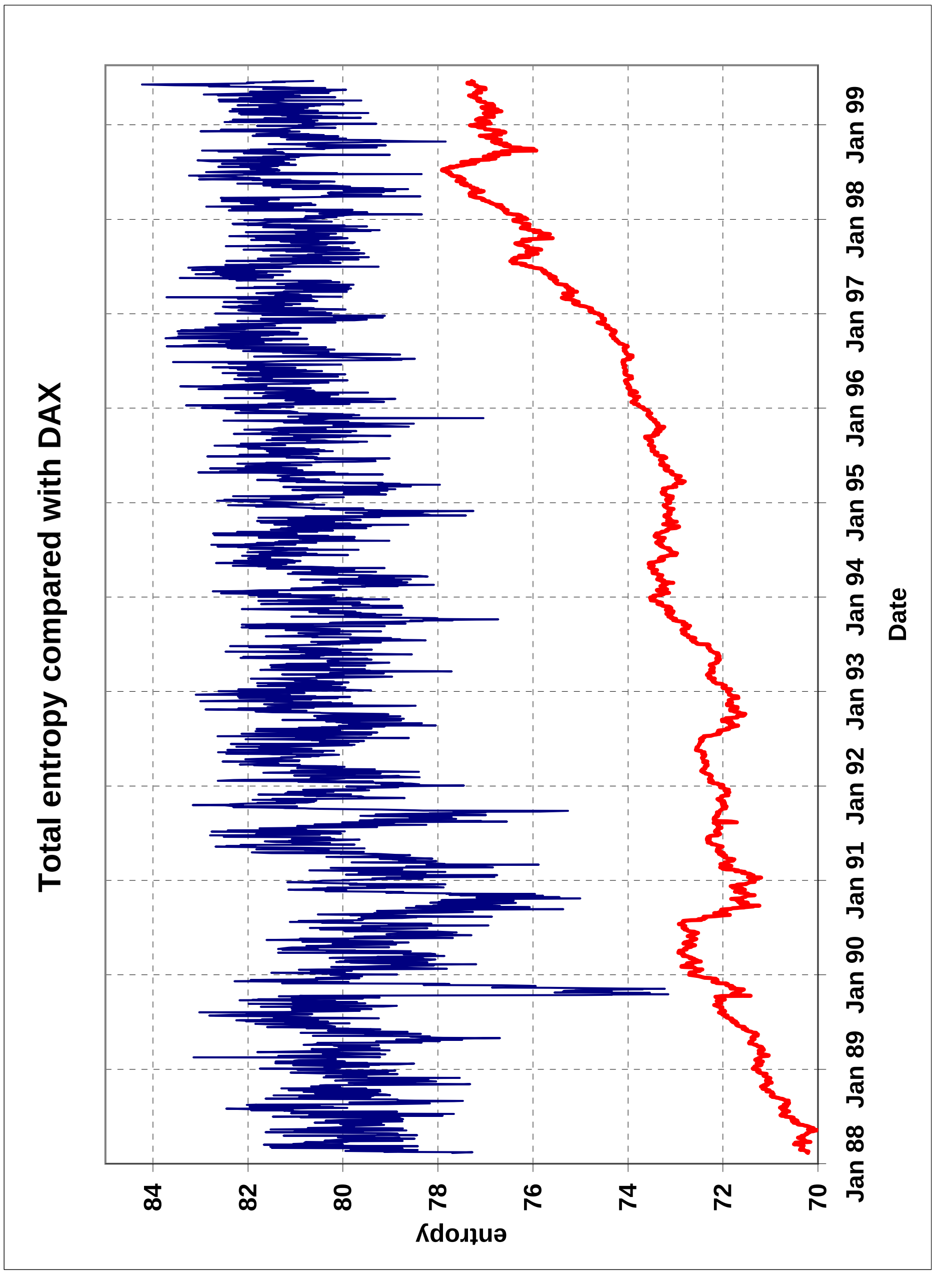

Figure 3: *Time-dependence of the total information entropy* I_{tot} *versus DAX.*

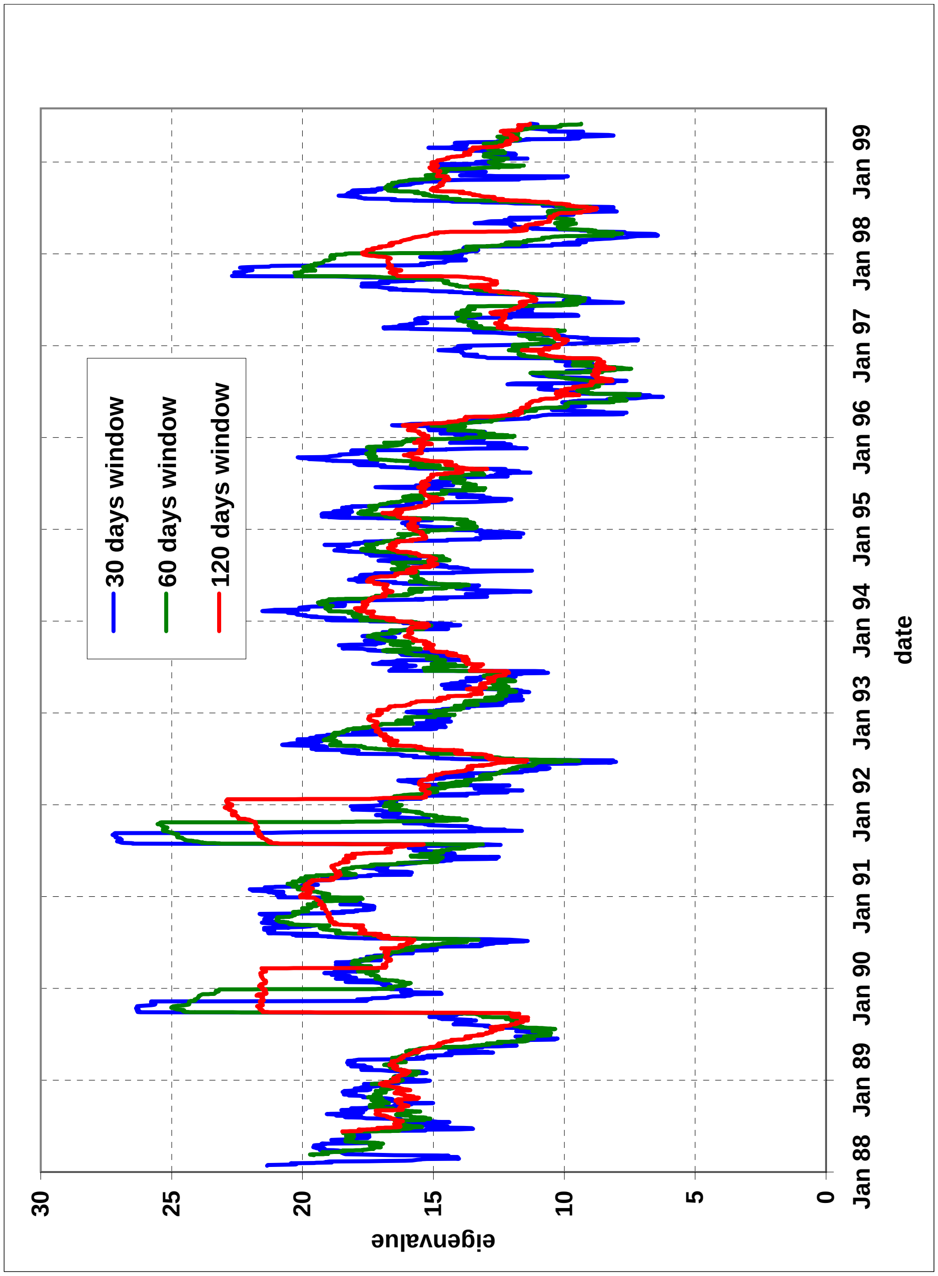

Figure 4: *Time-dependence of eigenspectra (as in Fig. 2) for* $T = 30$*, 60 and 120 days.*