SPT2002
Symmetry and Perturbation Theory
Cala Gonone, 19-26 May 2002

Abstracts of proposed talks

File last updated: 8/5/2002

Genrich Belitskii, Vadim Tkachenko
Functional moduli of local 1d diffeomorphisms

We consider local C^k diffeomorphisms of the real line with hyperbolic fixed points. It is well-known that if $k \geq 2$ then every such diffeomorphism is C^k conjugate with its linear part. For $k = 1$ the situation is different, and some functional moduli of classification arise. We describe these moduli and give some their applications.

Sergio Benenti
Some recent results on symmetry operators and separation of variables

The relationships between the additive separation of the Hamilton-Jacobi equation and the multiplicative (ordinary and conformal) separation of the associated Schrödinger equation and between first integrals and symmetry operators are examined within a geometrical framework. Suitable completeness, Robertson and pre-Robertson conditions are proposed.

Antonio Degasperis
Birth and properties of a new integrable PDE

The conjecture that a nonlinear (quadratic) partial differential equation of dispersive type is integrable was first announced at the SPT98 meeting. That conjectrure originates from necessary conditions for integrability based on a multiscale method. It has been now proved that that particular PDE is indeed integrable. We display its Lax pair and some of its properties and special solutions.

Florin Diacu
Qualitative properties of the anisotropic Manev problem

The anisotropic Manev problem is a two-body problem given by the Manev potential in an anisotropic space. Its main importance is that of finding connections between classical mechanics, quantum mechanics, and relativity. We will show that this problem has solutions that have classical, quantum, and relativistic properties.

Gregorio Falqui

Separation of variables and bihamiltonian geometry

We discuss some new results linking the theory of separation of variables for the Hamilton Jacobi equations with the theory of recursion operators and bihamiltonian structures for integrable systems. In particular, we formulate tensorial tests for separability, and apply such techniques to some classes of integrable models.

Sidi Fatimah

Suppressing flow-induced vibrations by parametric excitation

The possibility of suppressing self-excited vibrations of mechanical systems using parametric excitation is discussed. We consider a two-mass system of which the main mass is excited by a flow-induced, self excited force. A single mass which acts as a dynamic absorber is attached to the main mass and, by varying the stiffness between the main mass and the absorber mass, represents a parametric excitation. It turns out that for certain parameter ranges full vibration cancellation is possible. Using the averaging method the fully non-linear system is investigated producing as non-trivial solutions stable periodic solutions and tori. In the case of small absorber mass we have to carry out a second-order calculation.

Yuri Fedorov

Steklov-Lyapunov type systems and their integrable discretizations

Many finite-dimensional integrable systems (sometimes called *Gaudin magnets*) can be regarded as flows on finite-dimensional coadjoint orbits of the loop algebra $\widetilde{gl}(r)$ described by $r \times r$ Lax equations with a spectral parameter $\lambda \in \mathbf{C}$,

$$\dot{L}(\lambda) = [\, L(\lambda), \mathcal{M}(\lambda)\,], \qquad L = Y + (\mathcal{N}_i)/(\lambda - a_i), \quad L, \mathcal{M} \in gl(r),$$

where $Y \in gl(r)$ is a constant matrix and $a_1, \ldots, a_n$ are arbitrary distinct constants. These equations naturally arise in connection with so called rank r perturbations of the constant matrix $A = \mathrm{diag}(a_1, \ldots, a_n)$. In this talk we generalize this construction by introducing integrable flows on the space of three $n \times r$ matrices F, G, H of rank r that admit Lax pair of dimension $2r \times 2r$ with

$$L = Y + G^T(\lambda I_n - A)^{-1}(F + \lambda H),$$

where I_n is the $n \times n$ unit matrix. An appropriate reduction of these flows describe multidimensional generalizations of the Steklov and Lyapunov integrable cases in hydrodynamics, which were known before to have only a Lax pair with a (hyper)elliptic spectral parameter. We give a Hamiltonian description of the unreduced systems and present their discretizations preserving the first integrals.

Maria Luz Gandarias

Classical and nonclassical symmetry reductions of the Schwarz-KdV equation in (2+1) dimensions

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Guido Gentile

Renormalization group and summation of divergent series for hyperbolic invariant tori

The renormalization group methods for the resummations of perturbative series are applied to the study of hyperbolic low-dimensional invariant tori for a class of quasi-integrable analytic Hamiltonian systems. The perturbative series are shown to be analytical in the perturbative parameter inside a disc centered at the origin and deprived of a region around the positive or negative real axis with a quadratic cusp at the origin.

David Gomez Ullate

Quasi-exact solvability and Calogero-Sutherland models

We construct new families of Calogero-Sutherland models of particles with and without spin that are shown to be quasi-exactly solvable. The method uses an extension of the Dunkl operator approach and all the models obtained can be classified into equivalence classes. We obtain in paticular some examples of Calogero-Sutherland models with elliptic interaction for which some eigenvalues can be explicitly calculated.

Todor Gramtchev

Normal forms for vector fields and maps with linear parts having Jordan blocks

We will investigate convergent normal forms for analytic vector fields and maps with a fixed point with linear parts having nontrivial Jordan blocks.

Tamara Grava

Z_n curves Riemann-Hilbert problems and Schlesinger equations

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John Harnad

Matrix model integrals as Borel sums of Schur function expansions

The partition functions for unitary 2-matrix models are known to be KP τ-functions, as well as providing solutions to the 2-dimensional Toda hierarchy. It will be shown that they may also be viewed as Borel sum regularizations of divergent formal sums over products of Schur functions in the two sequences of associated KP flow variables. Two methods are used to derive this relation; one is based on reducing the M-dimensional matrix integral to the $M = 1$ case through a finite determinant identity; the second is a direct method based on a generating function expansion for Schur functions. This interpretation derives from new fermionic representations for matrix integrals. The element of the Grassmannian corresponding to the τ-function in either of two equivalent dual descriptions of the KP flows is identified in terms of the related bi-orthogonal polynomials. (Based on joint work with Alexandre Orlov.)

Alexander Kopanskii

Semi-local normal forms of vector fields

The talk deals with the problem of semi-local finitely smooth normalization of a vector field near an invariant submanifold. A condition providing C^k reduction of a C^K vector field ($k \leq K$) near a normally hyperbolic compact invariant manifold to a quasi-polynomial normal form is given.

Vadim Kuznetsov

Solution of inverse problem for integrable lattices

We develop a new technique that allows to solve the inverse problem for integrable lattices in the most explicit form, namely to describe a map from the linearizing (separation) variables to the initial (local) variables of an integrable lattice. Examples are given.

John McKay

Groups, finite, sporadic, discrete, and Lie and moonshine

The finite simple groups are essentially of Lie type or sporadic. The biggest sporadic group is the Monster, M, of order about 10^{54}. This group is related to discrete subgroups of PSL(2,R) and also the binary polyhedral groups, and through SL(2,5), to E8. There are many extraordinary facets of interest to physicists and connections with strings.

Ray McLenaghan

Group invariant classification of separable Hamiltonian systems in the Euclidean plane

An apparently new and effective method of determining separable coordinate systems for natural Hamiltonians in the Euclidean plane is presented. The method is based on intrinsic properties of the of the associated Killing tensors and their invariants under the group of rigid motions. Applications to the O(4)-symmetric Yang-Mills theories are considered. This is joint work with Roman Smirnov and Dennis The.

Alexander Mikhailov

Perturbative Symmetry Approach

We (me and Vladimir Nivikov) are developing a theory suitable for testing for integrability of a wide class of equations, including non-local, muti-dimensional and non-evolution equations. We illustrate our approach by examples of Benjamin-Ono and Camassa-Holm type equations. The approach is based on a perturbation theory and symbolic representation of the corresponding extension of a differential ring.

Nikolai Nekhoroshev

Types and mechanisms of integrability on phase space submanifolds

We discuss types and mechanisms of integrability on the phase space submanifolds N for vector fields of a general kind. We distinguish local, global, and torus integrability. The principal mechanism of local integrability is the existence of "sliding" Abelean symmetries of the given vector field V.

Sliding along the vector field V can also be defined for non-Abelian local symmetry Lie groups G. The sliding condition separates the submanifold N of the full space. This submanifold is automatically invariant with respect both to the group G and to the phase flow of the field V. The Abelian group $\mathcal{G} = \mathcal{G}(G, V)$ of sliding symmetries of V on N is constructed using the group G and the field V. If G is compact then V is a torically $\mathcal{G}$-integrable vector field on N.

Let G be just a local group. Suppose, however, that its "abelization" $\mathcal{G}$ has some special invariants on N. Then the torus $\mathcal{G}$-integrability on N is possible only on a submanifold $\mathcal{N} \subset N$. Such integrability is based on the Poincaré continuation of a compact orbit of the group $\mathcal{G}$. Parameters of this continuation are the values of the invariants of $\mathcal{G}$.

Peter Olver

Moving frames for pseudo-groups

Recent results on moving frames for infinite-dimensional Lie pseudo-groups

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Juan Pablo Ortega

The optimal momentum map and the geometry of Hamiltonian conservation laws

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Gianluca Panati

Space-adiabatic perturbation theory

A systematic perturbation scheme is developed to approximate solutions to the time-dependent Schroedinger equation with a "space-adiabatic" Hamiltonian. For a particular isolated energy band, we prove that interband transitions are suppressed to any order in epsilon. As a consequence, associated to that energy band there exists a subspace almost invariant under the unitary time evolution. We develop a systematic perturbation scheme for the computation of effective Hamiltonians which govern approximately the intraband time evolution. As examples for the general perturbation scheme we discuss the Dirac and Born-Oppenheimer type Hamiltonians. (Joint work with Herbert Spohn and Stefan Teufel)

Wilhelm Plesken

Janet's Algorithm

Janet's algorithm for linear partial differential equations is presented and various application are discussed e. g. for finding symmetries of differential equations. An implementation is available in a MAPLE package.

Emma Previato

Some integrable billiards

Poncelet's theorem on billiards inside the ellipse can be interpreted as a completely integrable Hamiltonian system and generalized to hyperelliptic curves of any genus.

Barbara Prinari

Discrete nonlinear Schroedinger systems

We study the IST for an integrable discretization of the matrix NLS equation. The vector NLS (VNLS), which is obtained as a reduction of a more general matrix system, arises, physically, under conditions similar to those described by NLS when the propagating field has two components transverse to the direction of propagation. The soliton solutions (vector solitons) are governed, both in the continuous and in the discrete case, by the classical NLS solitons but are characterized by a polarization and the vector nature of the solitons effects the dynamics of solitons with different polarization. We derive an analytic formula for the polarization shift of discrete vector solitons, which in the continuous limit reduces to the well-known Manakov's result for VNLS.

Giuseppe Pucacco

Separable systems in two dimensions as bi-Hamiltonian systems

We discuss the bi-Hamiltonian representation and Lax description of the dynamics of all separable two-dimensional systems using Jacobi metric and conformal transformations.

Tudor Ratiu

The analogue of the Euler-Poincaré equations in field theory

The Euler-Poincaré equations are the Lagrangian analogue of Hamilton's equations in Lie-Poisson bracket formulation. In this talk I shall give an overview how these equations are generalized to the field theoretical situation. Reduction for these equations will be also presented.

Stefan Rauch-Wojciechowski

Driven Newton equations and separation of time dependent potentials

The theory of quasipotential Newton equations provides a nontrivial generalisation of the classical separability theory for Hemholtz and Hamilton-Jacobi equations.

Study of quasipotential, driven equations $d^2y/dt^2 = M(y)$, $d^2x/dt^2 = M(y,x)$ leads to new type of separation for the Hamilton-Jacobi equation of the potential Newton equation of the form $d^2x/dt^2 = -\nabla V(y(t),x)$. A time dependent change of coordinates allows for the time variable to be separated of, leaving the remaining part in separable Stäckel form. We shall illustrate these results by a simple example.

Bob Rink

Direction reversing traveling waves in the even Fermi Pasta Ulam chain

We consider the Fermi Pasta Ulam chain with periodic boundary conditions and quartic nonlinearities. Due to special resonances and discrete symmetries, its Birkhoff normal form is a Liouville integrable approximation of this Hamiltonian system. Whereas the normal form equations can easily be solved if the number of particles in the chain is odd, they contain many nontrivial phenomena if the number of particles is even. In the latter case we observe that the phase space of the normal form is decomposed in subspaces that describe the interaction between modes with wave numbers j and $n/2 - j$. Moreover, we study how the level sets of the integrals of the normal form foliate the phase space. Our study reveals all the integrable structure in the low energy domain of the Fermi Pasta Ulam chain. The integrable foliation turns out to be singular and the method of singular reduction shows that the system has invariant pinched tori and monodromy. Monodromy is an obstruction to the existence of global action-angle variables. The pinched tori are interpreted as homoclinic and heteroclinic connections between traveling waves. Thus we discover a class of solutions which can be described as direction reversing traveling waves. They remarkably show an interesting interaction of the normal modes with wave numbers j and $n/2 - j$ in the absense of energy transfer. The direction reversing waves can easily be observed numerically in the original system.

Kjell Rosquist

The geometric approach to finite-dimensional integrable systems

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Andrea Sacchetti

Schroedinger equations with symmetric double-well potential under the effect of nonlinear perturbations

We prove that the beating motion, usually observed in a time-dependent Schroedinger equation with a symmetric double-well potential, disapears under the effect of a nonlinear perturbartion with strength larger than a critical value.

Jan Sanders

P-adic methods in integrability theory

abstract: P-adic numbers can be used to show the existence of a finite number of generalized symmetries of a given evolution equation. In this talk I will illustrate this with the proof of the existence of a system with two components that has exactly two symmetries, thereby invalidating an longstanding conjecture of Fokas that the existence of n symmetries for an n-component system implies the existence of infinitely many systems.

Paolo Santini

Initial-Boundary-Value Problems for Integrable PDEs

We discuss some spectral methods of solution of Initial- Boundary-Value problems for linear and nonlinear integrable PDEs. (Joint work with A.Degasperis and S.V.Manakov)

Roman Smirnov

Group invariants for separable Hamiltonians defined on Lorenzian spaces of zero curvature

We propose a new geometrical classification of separable Hamiltonian systems with two degrees of freedom defined in Minkowski space $\mathbf{R}^2_1$ admitting quadratic in momenta integrals of motion. The classification is based on group invariants of the associated Killing tensors.

Theo Tuwankotta

Slow-fast dynamics in system with widely separated frequencies

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Vladimir Tyuterev

Effective Hamiltonians and rovibrational quantum nuclear dynamics in molecules

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André Vanderbauwhede
Continuation and bifurcation of periodic orbits in symmetric Hamiltonian systems

We introduce the concept of normal periodic orbits (NPO) in symmetric Hamiltonian systems, and prove that such NPO belong to k-parameter families of NPOs, where k is the number of independent first integrals of the system. The approach used to prove this result can also be implemented numerically, leading to global bifurcation diagrams. We describe a few explicit examples in detail, and survey the results of an application of our method to the three-body-problem. This is joint work with E. Doedel, R. Paffenroth and J. Galan.

Ferdinand Verhulst
Behaviour near Slow Manifolds

Cecilia Vernia
Quasiperiodic Traveling Waves in Coupled Map Lattices

Study of Quasiperiodic Traveling Waves (QTW) in lattices of diffusively coupled logistic maps. Starting from the assumption that any spatial structure can be broken down into simpler elementary structures, a classification scheme for QTW is introduced. Within this framework, the phenomenon of discrete velocities is reviewed and presented. In addition, a new technique is proposed for predicting whether QTW can occur for given parameter values and which they might be.

Sebastian Walcher
Integrating factors of plane analytic systems

The task of determining a nontrivial infinitesimal symmetry of a plane (analytic) ordinary differential equation is equivalent to determining an integrating factor. The local (formal) integrating factors at elementary stationary points can be determined precisely, using normal form theory. This can be employed for certain global problems, and (via blow-ups) for non-elementary stationary points.

Jing Ping Wang
Integrable Systems in Riemannian Geometry

We show the evolution of the curvatures of an arc-lenght preserving curve in a 3-dimensional Riemannian manifold with constant curvature is determined by a Hamiltonian operator, which splits into four compatible Hamiltonian operators, giving rise to a number of integrable systems. However, this can not be generalized to n-dimensional Riemannian manifold. In the n-dimensional case, the parallel frame gives the Lax pair for vector modified KdV, from which we can derive the vector Sine-Gordon equation.

Pavel Winternitz

Superintegrable Classical and Quantum Systems

A review will be given of recent work on n-dimensional physical systems with more than n integrals of motion. Such systems have been studied for n=2 and 3, but also for n arbitrary. We shall consider flat configuration spaces as well as Riemannian spaces of variable, or constant curvature. We shall discuss integrals of motion that are second order in the momenta and also higher order ones.